

Dilaton Destabilization at High Temperature

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Abstract

Many compactifications of higher-dimensional supersymmetric theories have approximate vacuum degeneracy. The associated moduli fields are stabilized by non-perturbative effects which break supersymmetry. We show that at finite temperature the effective potential of the dilaton acquires a negative linear term. This destabilizes all moduli fields at sufficiently high temperature. We compute the corresponding critical temperature which is determined by the scale of supersymmetry breaking, the β -function associated with gaugino condensation and the curvature of the Kähler potential, $T_{\text{crit}} \sim \sqrt{m_{3/2} M_{\text{P}}} (3/\beta)^{3/4} K''^{-1/4}$. For realistic models we find $T_{\text{crit}} \sim 10^{11}\text{--}10^{12}$ GeV, which provides an upper bound on the temperature of the early universe. In contrast to other cosmological constraints, this upper bound cannot be circumvented by late-time entropy production.

1 Introduction

Compactifications of higher-dimensional supersymmetric theories generically contain moduli fields, which are related to approximate vacuum degeneracy. In many models these fields acquire masses through condensation of fermion pairs [1], which breaks supersymmetry. Generically, gaugino condensation models suffer from the dilaton ‘run-away’ problem [2], which can be solved, for example, by multiple gaugino condensates [3] or non-perturbative string corrections [4, 5].

Moduli play an important role in the effective low energy theory. Their values determine geometry of the compactified space as well as gauge and Yukawa couplings. Their masses, determined by supersymmetry breaking, are much smaller than the compactification scale. Hence, moduli can have important effects at low energies. Cosmologically, they can cause the ‘moduli problem’ [6, 7], in particular their oscillations may dominate the energy density during nucleosynthesis, which is in conflict with the successful BBN predictions. For an exponentially steep dilaton potential, like the one generated by gaugino condensation, there is also the problem that during the cosmological evolution the dilaton (S) may not settle in the shallow minimum at $\text{Re } S \sim 2$, but rather overshoot and run away to infinity [8]. These problems can be cured in several ways (cf. [9]).

In this paper we shall discuss a new cosmological implication of the dilaton dynamics, the existence of a critical temperature T_{crit} which represents an upper bound on allowed temperatures in the early universe. If exceeded, the dilaton will run to the minimum at infinity, which corresponds to the unphysical case of vanishing gauge couplings. The existence of a critical temperature is a consequence of a negative linear term in the dilaton effective potential which is generated by finite-temperature effects in gauge theories [10]. This shifts the dilaton field to larger values and leads to smaller gauge couplings at high temperature. As we shall see, this effect eventually destabilizes the dilaton, and subsequently all moduli, at sufficiently high temperatures. In the following we shall calculate the critical temperature T_{crit} beyond which the physically required minimum at $\text{Re } S \sim 2$ disappears.

There can be additional temperature-dependent contributions to the dilaton effective potential coming from the dilaton coupling to other scalar fields [11]. These contributions are model dependent and usually have a destabilizing effect on the dilaton, at least in heterotic string models [12]. Our results for the critical temperature can therefore be understood as conservative upper bounds on the allowed temperatures in the early universe.

The paper is organized as follows. In Sec. 2 we review the dependence of the free energy on the gauge coupling in $SU(N)$ gauge theories. As we shall see, one-loop corrections already yield the qualitative behaviour of the full theory. In Sec. 3 we study the dilaton potential at finite temperature and derive the critical temperature

T_{crit} for the most common models of dilaton stabilization. Sec. 4 is then devoted to the discussion of cosmological implications, the generality of the obtained results is discussed in Sec. 5, and the appendix gives some details on entropy production in dilaton decays.

2 Gauge couplings at high temperature

The free energy of a supersymmetric $\text{SU}(N_c)$ gauge theory with N_f matter multiplets in the fundamental representation reads

$$\mathcal{F}(g, T) = -\frac{\pi^2 T^4}{24} \left\{ \alpha_0 + \alpha_2 g^2 + \mathcal{O}(g^3) \right\}, \quad (1)$$

with g and T being the gauge coupling and temperature, respectively. The zeroth order coefficient, $\alpha_0 = N_c^2 + 2N_c N_f - 1$, counts the number of degrees of freedom, and the one-loop coefficient α_2 is given by (cf. [13])

$$\alpha_2 = -\frac{3}{8\pi^2} (N_c^2 - 1)(N_c + 3N_f). \quad (2)$$

It is very important that α_2 is negative. Hence, gauge interactions increase the free energy, at least in the weak coupling regime. Consequently, if the gauge coupling is given by the expectation value of some scalar field (dilaton) and therefore is a dynamical quantity, temperature effects will drive the system towards weaker coupling [10].

In reality, gauge couplings are not small, e.g., $g \simeq 1/\sqrt{2}$ at the GUT scale. Thus, higher order terms in the free energy are relevant. These could change the qualitative behaviour of the free energy with respect to the gauge coupling. For instance, in the case of a pure $\text{SU}(N_c)$ theory, the positive g^3 term overrides the negative g^2 term for $N_c \geq 3$. The knowledge of higher order terms is therefore necessary. These can be calculated perturbatively up to order $g^6 \ln(1/g)$, where the expansion in the coupling breaks down due to infrared divergences [13]. The non-perturbative contribution can be calculated by means of lattice gauge theory. For non-supersymmetric gauge theories with matter in the fundamental representation the free energy has been calculated up to $g^6 \ln(1/g)$ [14]. Comparison with numerical lattice QCD results shows that already the g^2 term has the correct qualitative behaviour, i.e., gauge interactions indeed increase the free energy. Furthermore, if terms up to order g^5 are taken into account, perturbation theory and lattice results are quantitatively consistent, even for couplings $g = \mathcal{O}(1)$ [14].

To demonstrate this behaviour, we consider the free energy of a non-supersymmetric gauge theory as a function of N_c and N_f using the results of Ref. [14] and earlier work [15]. As discussed, it is sufficient to truncate the perturbative expansion at order g^5 . We will be interested in the free energy in the

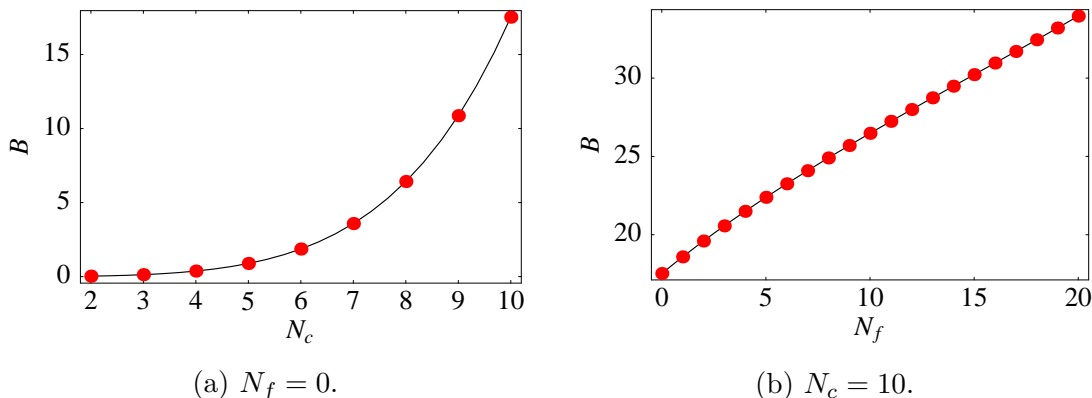


Figure 1: The coefficient B (cf. Eq. (3)) for $SU(N_c)$ gauge theory with N_f flavours; $g_0 = 1/\sqrt{2}$.

vicinity of a fixed coupling g_0 ,

$$\begin{aligned} g &= g_0 + \delta g, \\ \frac{\mathcal{F}(g, T)}{T^4} &= A(g_0) + B(g_0) \delta g + \mathcal{O}(\delta g^2). \end{aligned} \quad (3)$$

For our purposes, it is sufficient to keep the dominant linear term $\mathcal{O}(\delta g)$ and neglect higher order contributions $\mathcal{O}(\delta g^2)$, which have the same sign. Fig. 1(a) displays the coefficient B as a function of N_c with $N_f = 0$. Analogously, Fig. 1(b) shows the dependence of B on the number of matter multiplets N_f with $N_c = 10$. Obviously, B is positive and increases with the number of colours and flavours. This behaviour has to be the same for all non-Abelian gauge groups. The coefficient B will be even larger in supersymmetric theories due to gauginos and scalars.

3 Dilaton potential at finite temperature

In this section, we discuss how finite temperature effects modify the dilaton effective potential. This discussion applies to many string compactifications although details are model dependent. The major feature of the following analysis is that the dilaton potential has a minimum at $\text{Re } S \sim 2$ which is separated from another minimum at $\text{Re } S \rightarrow \infty$ by a finite barrier (see Fig. 2). This is a rather generic situation.

It is well known that gaugino condensation models generically suffer from the dilaton ‘run-away’ problem. That is, the minimum of the supergravity scalar potential is at $S \rightarrow \infty$, i.e., zero gauge coupling. The two most popular ways to rectify this problem in the framework of the heterotic string use multiple gaugino condensates [3, 16] and non-perturbative corrections to the Kähler potential [17, 18]. These mechanisms produce a local minimum at $\text{Re } S \sim 2$. As finite temperature effects due to thermalized gauge and matter fields drive the dilaton towards weaker coupling, this minimum can turn into a saddle point, in which case the dilaton would

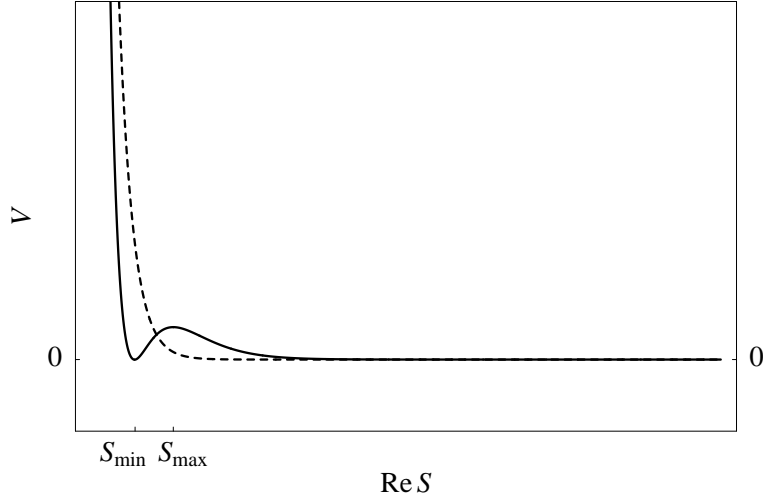


Figure 2: Typical potential for dilaton stabilization (solid curve). A minimum at $S = S_{\min} \simeq 2$ is separated from the other minimum at $S \rightarrow \infty$ by a finite barrier. For illustration, we also plot a typical run-away potential (dashed curve).

again run away. This puts a constraint on the allowed temperatures in the early universe.

If the hidden sector is thermalized (cf. [9]), such constraints are meaningful as long as the temperature is below the gaugino condensation scale, $\Lambda \sim 10^{13} - 10^{14}$ GeV. Otherwise, by analogy with QCD, it is expected that the gaugino condensate evaporates and the dilaton potential vanishes.

The critical temperature is obtained as follows. The stabilization mechanisms generate a local minimum of the dilaton potential at $\text{Re } S \sim 2$, immediately followed by a local maximum, with a separation $\delta \text{Re } S = \mathcal{O}(10^{-2})$. Beyond this local maximum, the potential monotonously decreases to the other minimum at $\text{Re } S \rightarrow \infty$. Since the dilaton interaction rate $\Gamma_S \sim T^3/M_{\text{P}}^2$ is much smaller than the Hubble parameter, the dilaton field is not in thermal equilibrium. It plays the role of a background field for particles with gauge interactions since its value determines the gauge coupling,

$$\text{Re } S = \frac{1}{g^2} . \quad (4)$$

As a consequence, the complete effective potential of the dilaton field is the sum of the zero-temperature potential V and the free energy \mathcal{F} of particles with gauge interactions,

$$V_T(\text{Re } S) = V(\text{Re } S) + \mathcal{F}(g = 1/\sqrt{\text{Re } S}, T) . \quad (5)$$

As the temperature increases, the local minimum and maximum of V_T merge into a saddle point at $\text{Re } S_{\text{crit}}$. This defines the critical temperature T_{crit} . $\text{Re } S_{\text{crit}}$ and

T_{crit} are determined by the two equations¹

$$V'(\text{Re } S_{\text{crit}}) + \mathcal{F}'(1/\sqrt{\text{Re } S_{\text{crit}}}, T_{\text{crit}}) = 0 , \quad (6)$$

$$V''(\text{Re } S_{\text{crit}}) + \mathcal{F}''(1/\sqrt{\text{Re } S_{\text{crit}}}, T_{\text{crit}}) = 0 , \quad (7)$$

where ‘prime’ denotes differentiation with respect to $\text{Re } S$.

We are only interested in the local behaviour of the potential around $\text{Re } S_{\text{min}} \simeq 2$, where we can expand the free energy $\mathcal{F}(g, T)$ as in Eq. (3) with

$$\delta g = -\frac{\delta \text{Re } S}{2(\text{Re } S_{\text{min}})^{3/2}} . \quad (8)$$

This produces a linear term in $\text{Re } S$ with a negative slope proportional to the fourth power of the temperature,

$$\mathcal{F}(g = 1/\sqrt{\text{Re } S}, T) = A T^4 - \delta \text{Re } S \frac{1}{\xi} T^4 + \mathcal{O}((\delta \text{Re } S)^2) , \quad (9)$$

where

$$\xi^{-1} = \frac{B}{2(\text{Re } S_{\text{min}})^{3/2}} . \quad (10)$$

Note, that validity of the linear approximation is based on the relation (4) between the gauge coupling and the dilaton field. In case of an arbitrary function $g = g(\text{Re } S)$ it does not necessarily hold.

In the linear approximation the equations for the critical value of the dilaton field and the critical temperature become (cf. (6), (7), (9)),

$$V''(\text{Re } S_{\text{crit}}) = 0 , \quad (11)$$

$$T_{\text{crit}} = \left(\xi V' \Big|_{\text{Re } S_{\text{crit}}} \right)^{1/4} . \quad (12)$$

At S_{crit} , which lies between S_{min} and S_{max} , the slope of the zero-temperature dilaton potential is maximal. It is compensated by the negative slope of the free energy at the critical temperature T_{crit} . For temperatures above T_{crit} the dilaton is driven to the minimum at infinity where the gauge coupling vanishes.

We can now proceed to calculating the critical temperature in racetrack and Kähler stabilization models. In what follows, we will assume zero vacuum energy, which can be arranged by adding a constant to the scalar potential. The hidden sector often contains non-simple gauge groups, e.g. in the case of nontrivial Wilson lines. Then gaugino condensation can occur in each of the simple factors [3]. Given the right gauge groups and matter content, the resulting superpotential can lead to dilaton stabilization at the realistic value of S [16]. For simplicity, we shall restrict ourselves to the case of two gaugino condensates.

¹In the case of more than one solution, the maximal T_{crit} is the critical temperature.

The starting point is the superpotential of gaugino condensation²,

$$W(S, \mathcal{T}) = \eta(\mathcal{T})^{-6} \Omega(S) , \quad (13)$$

where η is the Dedekind η -function and

$$\Omega(S) = d_1 \exp\left(-\frac{3S}{2\beta_1}\right) + d_2 \exp\left(-\frac{3S}{2\beta_2}\right) . \quad (14)$$

\mathcal{T} is the overall T-modulus parametrizing the size of the compactified dimensions. We assume that condensates form for two groups, $SU(N_1)$ and $SU(N_2)$, with M_1 and M_2 matter multiplets in the fundamental and anti-fundamental representations. The parameters d_i and the β -functions β_i are then given by ($i = 1, 2$),

$$\beta_i = \frac{3N_i - M_i}{16\pi^2} , \quad (15)$$

$$d_i = \left(\frac{1}{3}M_i - N_i\right) (32\pi^2 e)^{3(M_i - N_i)/(3N_i - M_i)} \left(\frac{1}{3}M_i\right)^{M_i/(3N_i - M_i)} . \quad (16)$$

Together with the Kähler potential

$$\mathcal{K} = K(S + \bar{S}) - 3 \ln(\mathcal{T} + \bar{\mathcal{T}}) , \quad (17)$$

the superpotential for gaugino condensation yields the scalar potential [16],

$$V = \frac{|\eta(\mathcal{T})|^{-12}}{(2 \operatorname{Re} \mathcal{T})^3} e^K \left\{ \frac{1}{K_{S\bar{S}}} |\Omega_S + K_S \Omega|^2 + \left(\frac{3(\operatorname{Re} \mathcal{T})^2}{\pi^2} |\hat{G}_2|^2 - 3 \right) |\Omega|^2 \right\} , \quad (18)$$

where subscripts denote differentiation with respect to the specified arguments, and the function \hat{G}_2 is defined via the Dedekind η -function as

$$\hat{G}_2 = - \left(\frac{\pi}{\operatorname{Re} \mathcal{T}} + 4\pi \frac{\eta'(\mathcal{T})}{\eta(\mathcal{T})} \right) . \quad (19)$$

It is well known that the T-modulus settles at a value $\mathcal{T} \sim 1$ in Planck units, independently of the condensing gauge groups [16]. Further, in the case of two condensates, minimization in $\operatorname{Im} S$ simply leads to opposite signs for the two condensates in Ω . From Eq. (18) we then obtain a scalar potential which only depends on $x \equiv \operatorname{Re} S$, the real part of the dilaton field,

$$V(x) = a e^K \left(\frac{4}{K''} \left(\Omega' + \frac{1}{2} K' \Omega \right)^2 - b \Omega^2 \right) , \quad (20)$$

where $a \simeq b \simeq 3$ and

$$\Omega(x) = d_1 \exp\left(-\frac{3x}{2\beta_1}\right) - d_2 \exp\left(-\frac{3x}{2\beta_2}\right) . \quad (21)$$

The dilaton is stabilized at a point x_{\min} where the first derivative of the potential,

$$\begin{aligned} V' = 2a e^K \left(\Omega' + \frac{1}{2} K' \Omega \right) & \left\{ \left(\Omega' + \frac{1}{2} K' \Omega \right) \left(4 \frac{K'}{K''} - 2 \frac{K'''}{K''^2} \right) \right. \\ & \left. + \frac{4}{K''} \Omega'' - \left(\frac{K'^2}{K''} + b - 2 \right) \Omega \right\} , \end{aligned} \quad (22)$$

²For simplicity, we neglect the Green-Schwarz term which would be an unnecessary complication in our analysis.

vanishes, and the dilaton mass term is positive,

$$m_S^2 = 2 \left. \frac{V''}{K''} \right|_{x_{\min}} > 0. \quad (23)$$

In the following we shall determine the critical temperature for two models of dilaton stabilization. The scales of dilaton mass and critical temperature are set by the gravitino mass,

$$m_{3/2}^2 = e^K |W|^2 \Big|_{x_{\min}} = a e^K |\Omega|^2 \Big|_{x_{\min}}, \quad (24)$$

and the scale of supersymmetry breaking, $M_{\text{SUSY}} = \sqrt{m_{3/2}}$, measured in Planck units.

3.1 Critical temperature for racetrack models

Consider first the case with the standard Kähler potential,

$$K(S + \bar{S}) = -\ln(S + \bar{S}), \quad (25)$$

and two gaugino condensates, the so-called ‘racetrack models’. The first derivative of the scalar potential (20) then becomes

$$V' = 2a e^K \left(\Omega' + \frac{1}{2} K' \Omega \right) \left(\frac{4}{K''} \Omega'' - (b-1) \Omega \right). \quad (26)$$

It has been shown [16] that the local minimum is determined by the vanishing of the first factor, $(2x \Omega'(x) - \Omega(x))|_{x_{\min}} = 0$.

We now have to evaluate (26) at the point of zero curvature, $V'' = 0$. Differentiation by x brings down a power of $3/(2\beta) \gg 1$. Away from the extrema, where cancellations occur, we therefore have the following hierarchy,

$$|\Omega| \ll |\Omega'| \ll |\Omega''| \ll |\Omega'''|. \quad (27)$$

This implies for the first and second derivative of the potential,

$$V' \simeq 2a e^K \frac{4}{K''} \Omega' \Omega'', \quad (28)$$

$$V'' \simeq 2a e^K \frac{4}{K''} (\Omega''^2 + \Omega' \Omega'''). \quad (29)$$

For the slope of the potential at the critical point one then obtains the convenient expression

$$\left. V' \right|_{x_{\text{crit}}} \simeq -2a e^K \frac{4}{K''} \frac{(\Omega')^2 \Omega'''}{\Omega''}. \quad (30)$$

For $x_{\min} < x < x_{\max}$ one has

$$\Omega' \sim -\frac{3}{2\beta_{\max}} \Omega, \quad \Omega''' \sim -\frac{3}{2\beta_{\min}} \Omega'', \quad (31)$$

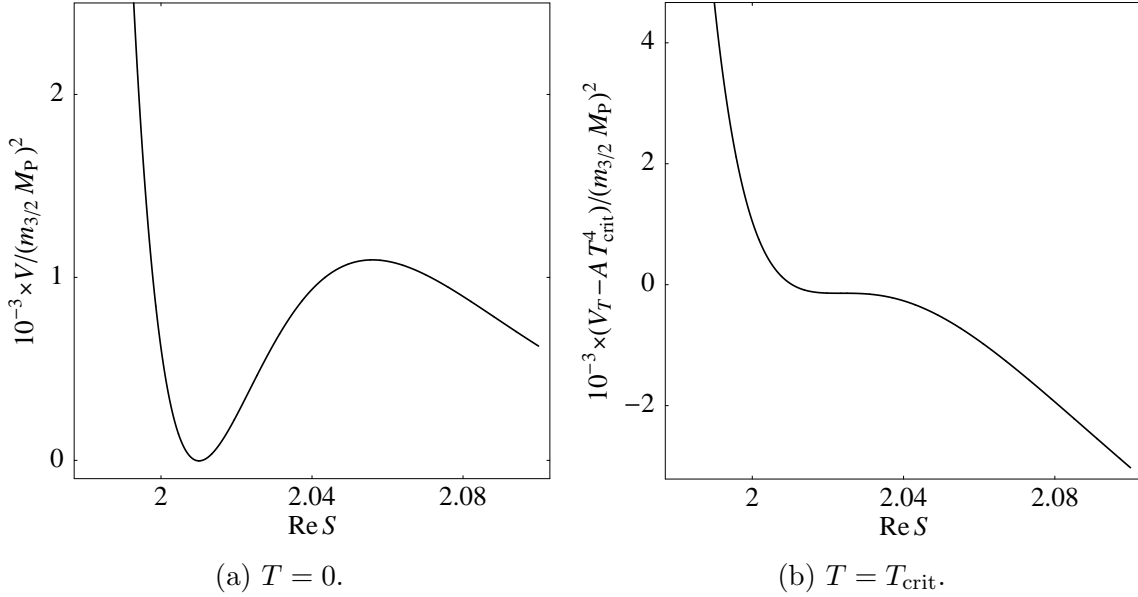


Figure 3: Dilaton potential for $(N_1, N_2) = (7, 8)$ and $(M_1, M_2) = (8, 15)$. (a): $T = 0$, (b): $T = T_{\text{crit}}$. In (b) the dilaton independent term $A T_{\text{crit}}^4$ has been subtracted (cf. Eq. (9)).

where β_{max} (β_{min}) is the larger (smaller) of the two β -functions. This yields for the slope of the potential

$$V'|_{x_{\text{crit}}} \sim 2a e^K \frac{4}{K''} \left(\frac{3}{2\beta_{\text{max}}} \right)^2 \left(\frac{3}{2\beta_{\text{min}}} \right) \Omega^2. \quad (32)$$

Since Ω does not vary significantly between x_{min} and x_{crit} , one finally obtains (cf. (24)),

$$V'|_{x_{\text{crit}}} \sim \frac{1}{K''} \left(\frac{3}{\beta_{\text{max}}} \right)^2 \left(\frac{3}{\beta_{\text{min}}} \right) m_{3/2}^2. \quad (33)$$

Using Eq. (12) we can now write down the critical temperature. Note that in racetrack models β_{min} and β_{max} are usually very similar. Introducing $\beta = (\beta_{\text{min}} \beta_{\text{max}}^2)^{1/3}$, one obtains

$$T_{\text{crit}} \sim \sqrt{m_{3/2}} \left(\frac{3}{\beta} \right)^{3/4} \left(\frac{\xi}{K''} \right)^{1/4}. \quad (34)$$

We have determined T_{crit} also numerically. The result agrees with Eq. (34) within a factor ~ 2 . The factor $\sqrt{m_{3/2}}$ appears since the scale of the scalar potential is set by $m_{3/2}^2$. The β -function factor corrects for the steepness of the scalar potential, whereas $(\xi/K'')^{1/4} = \mathcal{O}(1)$. With $m_{3/2} \sim 100 \text{ GeV}$, $\beta \sim 0.1$ and $M_{\text{P}} = 2.4 \times 10^{18} \text{ GeV}$, one obtains

$$T_{\text{crit}} \sim 10^{11} \text{ GeV}, \quad (35)$$

as a typical value of the critical temperature.

A straightforward calculation yields for the dilaton mass

$$m_S \simeq \frac{9}{\beta_1 \beta_2} \frac{1}{K''} m_{3/2} . \quad (36)$$

As a result, the dilaton mass is much larger than the gravitino mass and lies in the range of hundreds of TeV. This fact will be important for us later when we discuss the S -modulus problem.

3.2 Critical temperature for Kähler stabilization

As a second example we consider dilaton stabilization through non-perturbative corrections to the Kähler potential. In this case a single gaugino condensate is sufficient [17, 18]. Like instanton contributions, such corrections are expected to vanish in the limit of zero coupling and also to all orders of perturbative expansion. A common parametrization of the non-perturbative corrections reads

$$\begin{aligned} e^K &= e^{K_0} + e^{K_{\text{np}}} , \\ e^{K_{\text{np}}} &= c x^{p/2} e^{-q\sqrt{x}} , \end{aligned} \quad (37)$$

with $K_0 = -\ln(2x)$, $x = \text{Re } S$, and parameters subject to $K'' > 0$ and $p, q > 0$. For a single gaugino condensate, one has

$$\Omega = d \exp\left(-\frac{3x}{2\beta}\right) , \quad (38)$$

where $3/(2\beta) = 8\pi^2/N$ and $d = -N/(32\pi^2 e)$ for a condensing $\text{SU}(N)$ group with no matter. Note that the scalar potential is independent of $\text{Im } S$.

The scalar potential and its derivative are given by the simple expressions

$$V(x) = a e^K \Omega^2 \left(\frac{1}{K''} \left(K' - \frac{3}{\beta} \right)^2 - b \right) , \quad (39)$$

$$\begin{aligned} V'(x) &= a e^K \Omega^2 \left(K' - \frac{3}{\beta} \right) \left(\frac{1}{K''} \left(K' - \frac{3}{\beta} \right)^2 \right. \\ &\quad \left. - \frac{K'''}{(K'')^2} \left(K' - \frac{3}{\beta} \right) - b + 2 \right) . \end{aligned} \quad (40)$$

It has been shown [19, 17] that realistic minima are associated with the singularity at $K'' = 0$. That is, by tuning the parameters c, p, q it is possible to adjust $K'' = 0$ at some value x where the potential then blows up. By perturbing the parameters slightly, one obtains a finite potential with positive but small K'' , and the singularity smoothed out into a finite bump. The bump is located approximately at the point of minimal K'' , and the local minimum of the potential at $x \sim 2$ lies very close to it, with a separation $\delta x = \mathcal{O}(10^{-2})$.

For realistic cases, $K'(x \sim 2) \ll 3/\beta$, and the extrema of the potential around $x \sim 2$ are associated with the zeros of the last bracket in Eq. (40). As explained

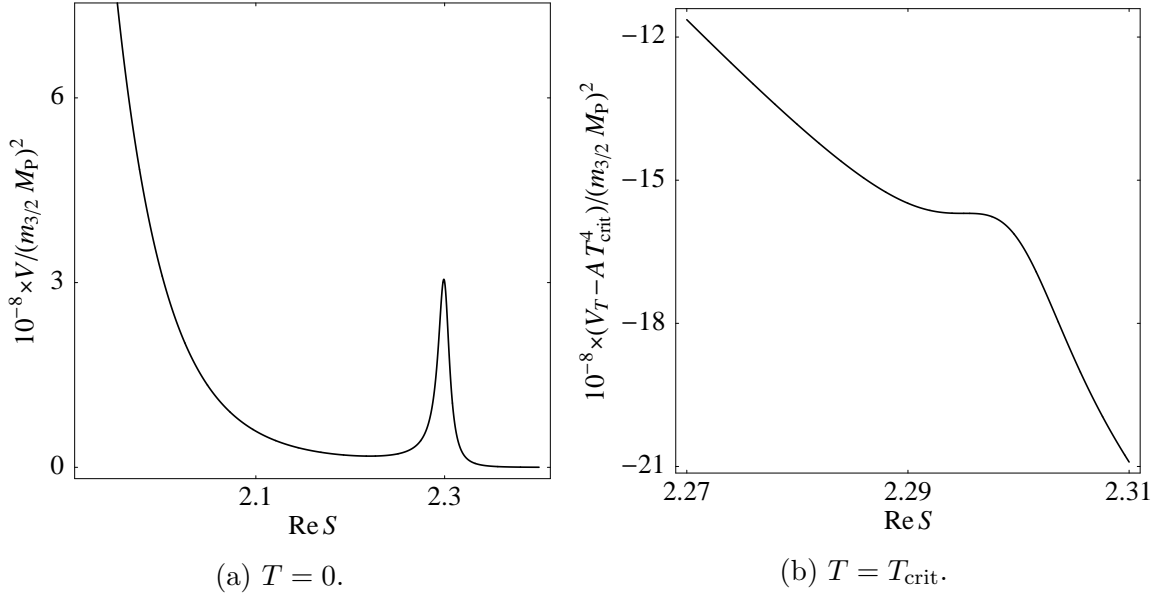


Figure 4: Dilaton potential for Kähler stabilization. $c = 5.7391$, $p = 1.1$, $q = 1$, and $N = 6$ [19]. (a): $T = 0$, (b): $T = T_{\text{crit}}$. In (b) the dilaton independent term $A T_{\text{crit}}^4$ has been subtracted (cf. Eq. (9)).

above, in practice K'' is a very small parameter such that one can expand in powers of K'' . Then, the extrema appear due to cancellations between the two ‘singular’ terms and we have the approximate relation

$$\frac{K'''}{K''} \simeq -\frac{3}{\beta}. \quad (41)$$

Due to the spiky shape of the potential, the point of vanishing curvature, $V'' = 0$, is very close to the local maximum. On the other hand, the cancellations between the $1/K''$ and $1/(K'')^2$ terms in Eq. (40) are not precise at this point and one can approximate their sum by the larger term. Using the fact that K and Ω do not vary significantly between x_{min} and x_{crit} , one obtains from Eqs. (24) and (41),

$$V'|_{x_{\text{crit}}} \sim a e^K \frac{1}{K''} \left(\frac{3}{\beta}\right)^3 \Omega^2 \sim \frac{1}{K''} \left(\frac{3}{\beta}\right)^3 m_{3/2}^2, \quad (42)$$

where K'' is evaluated at the local maximum x_{max} . Note that this result is identical to Eq. (33) which we have obtained for racetrack models. However, for these models $1/(K'')^{1/4} = \sqrt{x} = \mathcal{O}(1)$, whereas now K'' is a very small, but otherwise essentially free parameter.

Using Eq. (12) we find the same expression for the critical temperature as in racetrack models,

$$T_{\text{crit}} \sim \sqrt{m_{3/2}} \left(\frac{3}{\beta}\right)^{3/4} \left(\frac{\xi}{K''}\right)^{1/4}. \quad (43)$$

Since K'' is small in realistic models, the upper bound on allowed temperatures relaxes compared to racetrack models. As before, Eq. (43) agrees within a factor ~ 2 with numerical results. A typical value of the critical temperature is obtained for $m_{3/2} \sim 100$ GeV, $\beta \sim 0.1$ and $K'' \sim 10^{-4}$,

$$T_{\text{crit}} \sim 10^{12} \text{ GeV} . \quad (44)$$

For the dilaton mass one obtains

$$m_S \sim \left(\frac{3}{\beta}\right)^2 \frac{1}{K''} m_{3/2} . \quad (45)$$

Again, we find that the dilaton is much heavier than the gravitino.

4 Implications for cosmology

As we have seen in the previous section, the dilaton gets destabilized at high temperature. The maximal allowed temperature is given by $T_{\text{crit}} \sim 10^{11} - 10^{12}$ GeV. In this section, we study implications of this bound for cosmology.

Most importantly, T_{crit} represents a model independent upper bound on the temperature of the early universe,

$$T < T_{\text{crit}} . \quad (46)$$

This bound applies to a large class of theories, with weakly coupled heterotic string models being the most prominent representatives. It is worth emphasizing that the dilaton destabilization effect is qualitatively different from the gravitino [20] or moduli problems [6, 7] in that it cannot be circumvented by invoking other effects in late-time cosmology such as additional entropy production. Once the dilaton goes over the barrier, it cannot come back.

The present bound applies to any radiation dominated era in the early universe, even if additional inflationary phases occur afterwards. Therefore, T_{crit} not only provides an upper bound on the reheating temperature T_R of the last inflation, but also can be regarded as an absolute upper bound on the temperature of the radiation dominated era in the history of the universe.

4.1 S -modulus problem and thermal leptogenesis

In addition to the bound discussed above, one can have further, more model dependent, constraints on temperatures occurring at various stages of the evolution of the universe. In this subsection, we discuss one of them, related to the S -modulus problem.

Even if the reheating temperature does not exceed the critical one, thermal effects shift the minimum of the dilaton potential. Due to this shift, S starts coherent oscillations after reheating. Since the energy density stored in the oscillations

behaves like non-relativistic matter, $\rho_{\text{osc}} \propto R^{-3}$, with R being the scale factor, it grows relative to the energy density of the thermal bath, $\rho_{\text{rad}} \propto R^{-4}$, until S decays. Its lifetime can be estimated as $(\Gamma_S)^{-1} \sim M_{\text{P}}^2/m_S^3 \simeq 0.004 \text{ s } (m_S/100 \text{ TeV})^{-3}$. In the examples studied in Sec. 3, $m_S \gg 10 \text{ TeV}$, so that S decays before BBN. Thus, there is no conventional moduli problem, i.e., dilaton decays do not spoil the BBN prediction of the abundance of light elements.

However, even for these large masses, coherent oscillations of S may affect the history of the universe via entropy production [6, 7]. Let us estimate the initial amplitude of these oscillations. At a given temperature $T \ll T_{\text{crit}}$, the dilaton potential around the minimum can be recast as

$$V_T = \frac{1}{2} m_S^2 \phi^2 - \sqrt{\frac{2}{\xi^2 K''}} T^4 \frac{\phi}{M_{\text{P}}}, \quad (47)$$

where $\phi = M_{\text{P}} \sqrt{K''/2} \text{Re}(S - S_{\text{min}})$. The minimum of the potential is at

$$\langle \phi \rangle_T \simeq \sqrt{\frac{2}{\xi^2 K''}} \frac{T^4}{m_S^2 M_{\text{P}}}. \quad (48)$$

Thus, at $T = T_R$, the displacement of ϕ from its zero temperature minimum is estimated as $\Delta \phi|_{T_R} \sim \langle \phi \rangle_{T_R}$. Then, the entropy produced in dilaton decays is (see Appendix),

$$\Delta = \frac{s_{\text{after}}}{s_{\text{before}}} \sim \frac{1}{\xi^2 K''} \left(\frac{T_R}{10^{10} \text{ GeV}} \right)^5 \left(\frac{10^6 \text{ GeV}}{m_S} \right)^{7/2}. \quad (49)$$

The decay occurs at temperatures of order 10 MeV, i.e., after the baryon asymmetry and the dark matter abundance have been fixed. Thus, we see that for $T_R \gtrsim 10^{10} \text{ GeV } (m_S/10^6 \text{ GeV})^{7/10} (\xi^2 K'')^{1/5}$, the baryon asymmetry and relic dark matter density get significantly diluted.

For instance, successful thermal leptogenesis [21] requires $T_R \gtrsim T_{\text{L}} \simeq 3 \times 10^9 \text{ GeV}$ [22]. For $T_R \gtrsim T_{\text{L}}$, the baryon asymmetry can be enhanced by T_R/T_{L} , but later it gets diluted by a factor $\propto T_R^5$. Hence, there is only a narrow temperature range where thermal leptogenesis is compatible with the usual mechanisms of dilaton stabilization. We note further that, in this range of temperatures, the bound on the light neutrino masses tightens. For instance, $T_R < 3 \times 10^{10} \text{ GeV}$ implies³ $m_i \lesssim 0.07 \text{ eV}$, which is more stringent than the temperature-independent constraint, $m_i \lesssim 0.1 \text{ eV}$ [24].

Concerning dark matter, we note that in WIMP cold dark matter scenarios, at the time of the dilaton decay the pair annihilation processes have frozen out so that the entropy production reduces Ω_{CDM} .⁴ This effect could be welcome in parameter regions where otherwise WIMPs are overproduced. Entropy production could also contribute to the solution of the gravitino problem.

³Here we have used Fig. 10 of Ref. [22], $m_1 < \tilde{m}_1$ [23], and $m_3^2 - m_1^2 \simeq \Delta m_{\text{atm}}^2$.

⁴WIMP dark matter may be directly produced by moduli decay [25].

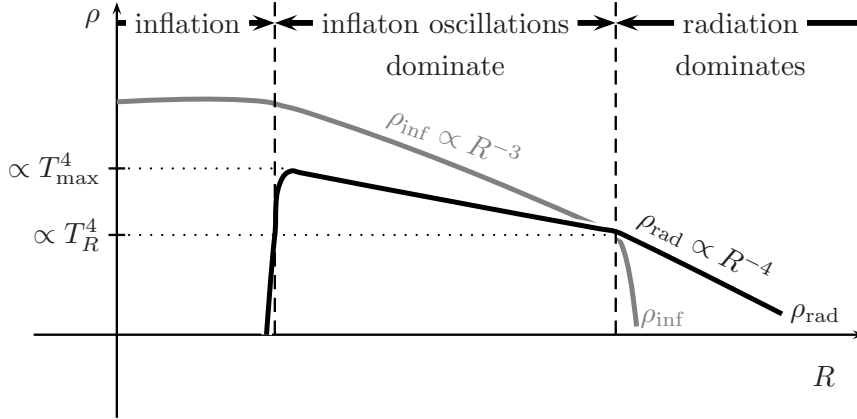


Figure 5: Three epochs in inflationary models: inflation, inflaton oscillation domination and radiation domination [26].

It is important to remember that the T-moduli problem remains. Thermal effects shift all moduli from their zero temperature minima, thereby inducing their late coherent oscillations. Unlike the dilaton, other moduli typically have masses of order $m_{3/2}$ and thus tend to spoil the BBN predictions.

In summary, there exists a range of reheating temperatures, $10^{-2}T_{\text{crit}} \lesssim T_R \lesssim T_{\text{crit}}$, which are cosmologically acceptable, but for which the history of the universe is considerably altered, in particular via significant entropy production at late times.

4.2 Further constraints on inflation models

In this subsection we discuss some implications of the thermal effects at earlier times, before the reheating process completes. There are three important stages in the inflationary scenario: inflation, the inflaton–oscillation epoch, and the radiation dominated epoch (see Fig. 5).

During inflation, the energy density of the universe is dominated by the potential energy of the inflaton χ . After the end of inflation, inflaton starts its coherent oscillations. The energy density of the universe is still dominated by the inflaton χ , until the reheating process completes and radiation energy takes over with temperature $T = T_R$. The nonzero energy density of the inflaton induces additional SUSY breaking effects [27]. Hence, one may expect that the dilaton potential is also affected by the finite energy of the inflaton χ during these χ -dominated eras.

Further, in the χ -oscillation era there is radiation with temperature $T \simeq (T_R^2 M_{\text{P}} H)^{1/4}$ [26], where H is the Hubble parameter. Although its energy density is small compared to that of inflaton (see Fig. 5), it affects the dilaton potential as we have discussed in Sec. 3. Since the maximum temperature T_{max} in the χ -oscillation era is generically higher than the reheating temperature T_R , one expects stronger constraints from $T_{\text{max}} < T_{\text{crit}}$.

Whether it is radiation or inflaton that affects the dilaton potential more, depends on the coupling between dilaton and inflaton. As this is model dependent, below we consider the three possible cases:

(i) destabilizing dilaton–inflaton coupling. The inflaton–dilaton coupling drives the dilaton to larger values and may let it run away to infinity. This puts severe constraints on the inflation model. Some models can be excluded independently of the reheating temperature.

(ii) stabilizing dilaton–inflaton coupling. The inflaton effects move the dilaton to smaller values. In this case, the previously obtained bound on the reheating temperature $T_R < T_{\text{crit}}$ provides the most stringent constraint. Note that the shift of the dilaton may cause a large initial amplitude of its oscillation, which can result in a late-time entropy production as discussed in Sec. 4.1.

(iii) negligible dilaton–inflaton coupling. In this case, the effect of radiation during the χ -oscillation era (preheating epoch) is dominant. The maximal radiation temperature can be expressed in terms of the reheating temperature [26],

$$T_{\text{max}} \simeq (T_R^2 M_{\text{P}} H_{\text{inf}})^{1/4}, \quad (50)$$

where H_{inf} is the Hubble expansion rate during inflation. T_{max} must be below the critical temperature, or the dilaton will run away to weak coupling. This constraint translates into a bound on the reheating temperature depending on H_{inf} ,

$$T_R \lesssim \left(\frac{T_{\text{crit}}^4}{M_{\text{P}} H_{\text{inf}}} \right)^{1/2} \simeq 6 \times 10^7 \text{ GeV} \left(\frac{T_{\text{crit}}}{10^{11} \text{ GeV}} \right)^2 \left(\frac{10^{10} \text{ GeV}}{H_{\text{inf}}} \right)^{1/2}, \quad (51)$$

as shown in Fig. 6. The upper bound on T_R now becomes much severer. For instance, taking $T_{\text{crit}} \simeq 10^{11} \text{ GeV}$ and typical values of H_{inf} in some inflation models⁵ (cf. [29]), we obtain the following bounds:

$$\begin{aligned} \text{chaotic inflation:} \quad & H_{\text{inf}} \simeq 10^{13} \text{ GeV}, \quad T_R \lesssim 10^6 \text{ GeV}, \\ \text{hybrid inflation:} \quad & H_{\text{inf}} \sim 10^8\text{--}10^{12} \text{ GeV}, \quad T_R \lesssim 10^7\text{--}10^9 \text{ GeV}, \\ \text{new inflation:} \quad & H_{\text{inf}} \sim 10^6\text{--}10^{12} \text{ GeV}, \quad T_R \lesssim 10^7\text{--}10^{10} \text{ GeV}. \end{aligned}$$

These bounds apply if already during the preheating phase particles with gauge interactions form a plasma with temperature T_{max} and the dilaton is near the physical minimum.

5 Conclusions

At finite temperature the effective potential of the dilaton acquires locally a negative linear term. As we have seen, this important fact is established beyond perturbation theory by lattice gauge theory results. As a consequence, at

⁵In curvaton scenarios [28] the values of H_{inf} are much less constrained.

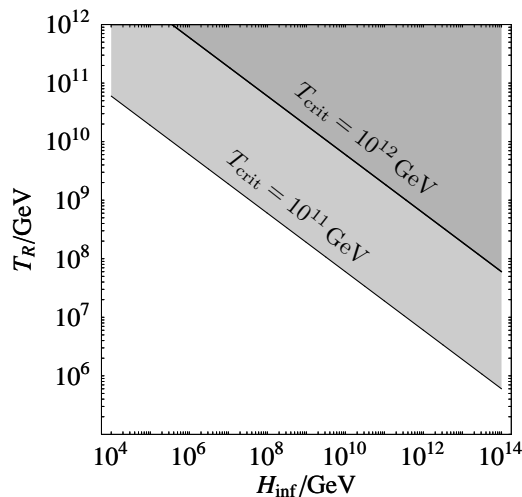


Figure 6: Upper bounds on the reheating temperature for $T_{\text{crit}} = 10^{11}$ GeV and 10^{12} GeV assuming a small inflaton–dilaton coupling (case (iii)). The darker region is excluded for $T_{\text{crit}} = 10^{12}$ GeV; for $T_{\text{crit}} = 10^{11}$ GeV the lighter region is excluded as well.

sufficiently high temperatures the dilaton S , and subsequently all other moduli fields, are destabilized and the system is driven to the unphysical ground state with vanishing gauge coupling. We have calculated the corresponding critical temperature T_{crit} which is larger than the scale of supersymmetry breaking, $M_{\text{SUSY}} = \sqrt{M_{\text{P}} m_{3/2}} = \mathcal{O}(10^{10} \text{ GeV})$, but significantly smaller than the scale of gaugino condensation, $\Lambda = [d \exp(-3S/(2\beta))]^{1/3} M_{\text{P}} = 10^{13}\text{--}10^{14} \text{ GeV}$. This is the main result of our paper.

Our result is based on the well understood thermodynamics of the observable sector. In contrast, the temperature of gaugino condensation can place a bound on the temperature of the early universe only under the additional assumption that the hidden sector is thermalized.

The upper bound on the temperature in the radiation dominated phase of the early universe, $T < T_{\text{crit}} \sim 10^{11}\text{--}10^{12} \text{ GeV}$, has important cosmological implications. In particular, it severely constrains baryogenesis mechanisms and inflation scenarios. Models requiring or predicting $T > T_{\text{crit}}$ are incompatible with dilaton stabilization. In contrast to other cosmological constraints, this upper bound cannot be circumvented by late-time entropy production.

We have also discussed more model dependent cosmological constraints. Even if $T < T_{\text{crit}}$, the S -modulus problem restricts the allowed temperature of thermal leptogenesis and makes the corresponding upper bound on light neutrino masses more stringent. Furthermore, depending on the assumed coupling between dilaton and inflaton, stronger bounds can apply to the reheating temperature.

Our discussion has been based on the assumption that moduli are stabilized by non-perturbative effects which break supersymmetry. Thus the barrier separating

the realistic vacuum from the unphysical one with zero gauge couplings is related to supersymmetry breaking. Recently, an interesting class of string compactifications has been discussed where fluxes lead to moduli stabilization and supersymmetry breaking (see, for example, [30, 31, 32]). Realistic, metastable de Sitter vacua also require non-perturbative contributions to the superpotential from instanton effects or gaugino condensation [31]. It remains to be seen how much fluxes can modify the critical temperature in realistic string compactifications.

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Appendix Evolution of ϕ and entropy production

In the following, we derive the dilution factor, Eq. (49). The dilaton starts coherent oscillations soon after the radiation dominated era begins. This is because the effect of the temperature term in Eq. (47) disappears very quickly and when $|\phi - \langle\phi\rangle_T| \gtrsim \langle\phi\rangle_T$ the potential becomes essentially quadratic. As can be verified numerically, the initial amplitude of subsequent oscillations is close to the initial displacement of the dilaton from its zero temperature minimum, $\Delta\phi|_{T_R} \sim \langle\phi\rangle_{T_R}$. The Hubble friction is very small at these times, $H \ll m_S$.

The ratio of ρ_{osc} to the entropy density s before the dilaton decays is given by (cf. Eq. (48)),

$$\left. \frac{\rho_{\text{osc}}}{s} \right|_{\text{before}} \sim \frac{m_S^2 \langle\phi\rangle_{T_R}^2}{s(T_R)} = \frac{2T_R^5}{(2\pi^2/45) g_*(T_R) \xi^2 K'' m_S^2 M_{\text{P}}^2} . \quad (52)$$

The ratio stays constant since $\rho_{\text{osc}} \propto s \propto R^{-3}$. Just after the dilaton decays, the ratio of ρ_{rad} to s is

$$\left. \frac{\rho_{\text{rad}}}{s} \right|_{\text{after}} = \frac{3}{4} T_d \simeq \frac{3}{4} \left(\frac{\pi^2}{90} g_*(T_d) \right)^{-1/4} \sqrt{\Gamma_S M_{\text{P}}} . \quad (53)$$

If $\rho_{\text{osc}}/s > \rho_{\text{rad}}/s$, the dilaton decay causes large entropy production. Using $\rho_{\text{rad}}|_{\text{after}} \simeq \rho_{\text{osc}}|_{\text{before}}$, we obtain Eq. (49). Note that there are large numerical uncertainties in this expression due to the dependence on initial conditions. In extreme scenarios, Δ can be close to one. However, the resulting uncertainty in T_R is usually rather small since it appears with the fifth power.

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